

Wave Refraction across a Current with Strong Horizontal Shearing

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Abstract

Experiments have been performed with regular deep-water waves propagating obliquely across a relatively narrow jet-type current. Measurements have been made of wave refraction using an 8-element wave array and 3d acoustic Doppler flowmeters. Results show that wave height and celerity follow the trends predicted by gradually-varied flow theory, but with wave height modulated by weak reflections as the waves pass across the jet. Direction of wave propagation is found to be sensitive to the presence of internal reflections from the shear layers.

Introduction

It is now widely accepted that non-linear wave-current interaction plays an important role in the evolution of coastal flow. Currents are generated by wind stresses, tides and river estuary discharges, and when waves move across such spatially varying mean flows, they experience significant changes in amplitude, celerity, direction, kinematics, and bed friction, all of which affect both their local characteristics and their potential impact on the wider coastal environment.

Many theoretical models of coastal flows are based on the assumption that waves are moving through a gradually varying medium, for instance, where there is a gently sloping seabed or a current contains only weak horizontal shearing. Conventionally, when regular long-crested waves cross a horizontally sheared current, the angle of refraction and Doppler-shifted celerity are determined by application of Snell's Law to the wave orthogonals, while the change in wave height

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is determined from the conservation of wave action through wave ray tubes (Jonsson, 1990). This form of analysis is appropriate when the width of the shear layer is many times the length of the incident wave, such as where waves propagate across the Gulf Stream. In that particular case, the width of the jet current is approximately 300 times the incident wave length.

However, it is often the case that waves propagate through waters where bathymetry and currents vary rapidly, for instance, where rivers or tidal inlets discharge into the ocean. Under such circumstances the width of the jet may be less than 5 times the incident wave length. An important consequence of such rapidly varying conditions is that significant wave reflection may take place at both shear layers, a phenomenon not normally included in wave refraction models.

There have been a number of attempts to model wave refraction across a rapidly varying current, extending the slowly-varying WKB solution to allow reflections (McKee, 1977; McKee, 1996), or including a vortex sheet to simulate the reflecting shear layer (Smith, 1983; Kirby, 1986). The availability of data against which these theories can be validated has been restricted by the practical difficulties involved in performing equivalent experiments. At a laboratory scale the flow conditions typically achievable can produce refraction of only a few degrees, and the accurate measurement of such small changes presents a significant difficulty. However, motivated by a parallel project at University College Cork to develop a new model, the present paper describes tests carried out under controlled conditions in the UK Coastal Research Facility (UKCRF) at Wallingford aimed at providing both a better understanding of the processes controlling current refraction and data for model validation.

The experiments were designed to investigate the behaviour of waves in water of constant depth as they pass obliquely across a narrow jet-like current with strong horizontal shear. The refraction has been established at several locations across the shore-parallel current from oscillatory velocity vectors and from a directional wave array. The project addresses one of the areas highlighted in a recent review (Thomas and Klopman, 1997) as requiring attention, and offers a unique data set against which models can be tested.

Experiments

The UKCRF has been designed to provide a controlled environment in which various coastal processes can be simulated and investigated at relatively large scale. While the main aim is to provide data for validation of models describing a range of coastal phenomena, it also offers the opportunity to improve fundamental understanding of the physics. The design and capabilities of the basin are described in an earlier paper (Simons *et al.*, 1995).

For the present tests, water depth was fixed at 0.5m above a bed roughened with 10mm diameter granite chippings. A jet-like current was generated in the offshore flat-bed region of the basin. This jet had a Gaussian horizontal distribution of velocity across its nominal width of 7 metres and a streamwise centreline near-surface velocity of 0.25m/s. Long-crested waves, generated in still water offshore from the jet current, were propagated through the jet current before being spent on a 1-in-20 plane beach. Preliminary tests used regular wave periods of both 0.8s and 0.9s, wave heights of 40mm, and angles of incidence between wave orthogonal and shore-normal of 0° and ±30°. Parameters for the main test programme described here are set out in **Table 1** below.

UK Coastal Research Facility	
Basin dimensions:	36m by 20m (internal)
Test region:	10m alongshore by 9m shore-normal
Water depth:	0.5m
Bed geometry:	horizontal
Bed roughness:	10mm diameter (nominal) stone chippings
Spending beach:	1 in 20 slope (outside test region)
Jet Current	
Centreline velocity:	0.25 m/s at test section
Width:	7.0m (4.0m for $\bar{u}_{50}$) at test section
Horizontal profile:	Gaussian
Flow direction:	shore-parallel
Waves	
Regular wave period:	0.8s
Height:	0.04m (nominal)
Propagation direction:	-30°, 0°, 30°
h / L	0.50
Prototype scale:	
Depth:	25m
Centreline velocity:	1.75 m/s
Wave period:	6s
Width of jet:	200m

Table 1: Test conditions

Vertical profiles of mean and wave-induced velocities were measured at regular positions across and at three sections along the horizontally varying current using three 3d Nortek acoustic Doppler velocimeters (ADV) mounted one above the other. Surface elevation was measured using the “Octoprobe”, an array of eight wave probes mounted in a fixed geometry, 0.85m long by 0.6m wide, as described by

Teisson and Benoit (1994). The array was traversed across the jet current with one wave probe located exactly above the three ADV velocimeters, and provided wave heights, reflection coefficients and wave propagation direction. Four additional wave probes were also deployed on tripods, at fixed locations in the basin to provide a common reference for phase analysis of all data channels.

The objectives of the test programme were: with no waves present, to establish a stable jet current parallel to the wave generators, and to measure the velocity field across the jet and through the depth at three cross-sections; then, with no jet current flowing, to measure the velocity field for regular long-crested waves propagating obliquely or normal to the shoreline; and finally, with those waves propagating across the jet current, to measure the wave refraction and wave height at regular positions across the jet current, and to determine the Doppler-shifted wave celerity at the same locations.

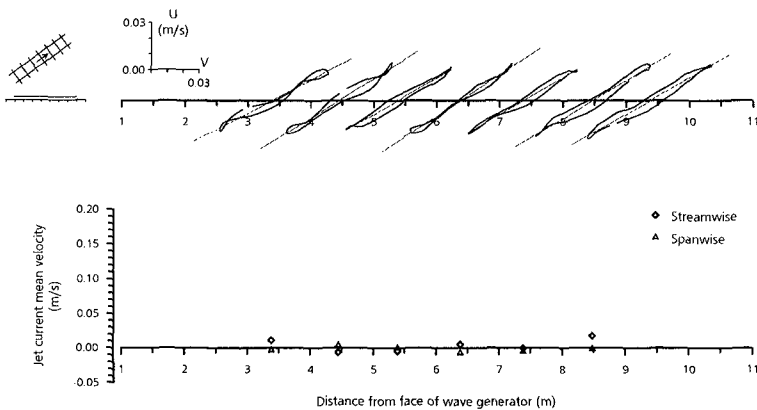


Figure 1: Horizontal velocity vectors for 30° waves: no current flowing [straight lines indicate mean direction of wave propagation].

Methods of analysis

To determine the local direction of wave propagation as the waves refract across the jet current, five different methods of analysis were considered.

The first of these ("Method 1") used only data from the ADV velocity probes to generate the ensemble average horizontal velocity vector at phases through the wave cycle. A least-squares method was then adopted to identify the mean direction

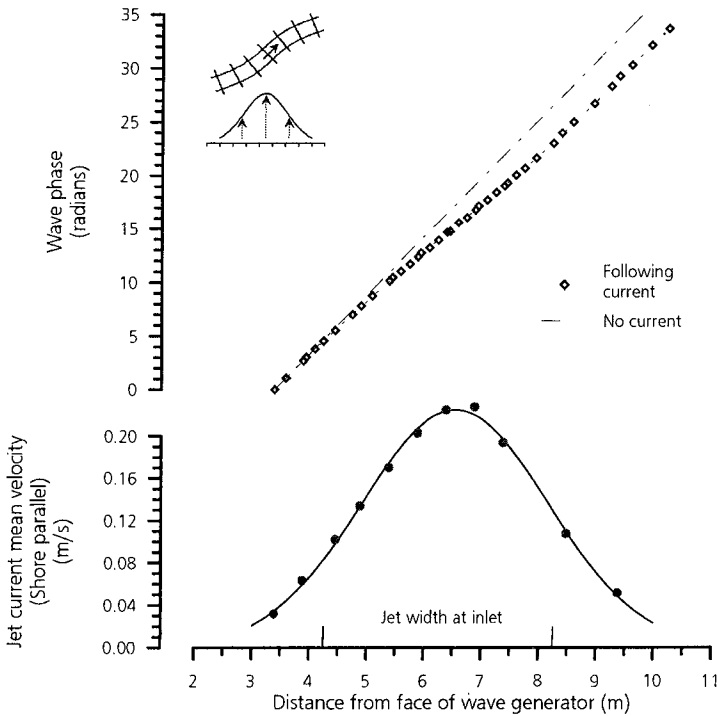


Figure 2: Variation in wave phase on an orthogonal section across the jet current.

of the velocity vector. An alternative method, calculating the direction using only the 1st harmonic of the horizontal velocity vector, yielded almost identical results. It should be noted that the vectors were never the ideal straight lines that a clean, long-crested wave should produce, but were always mildly contorted closed curves, as shown in **fig.1** for a case with no current. This suggests that waves of different frequency or direction of propagation were present in the basin.

“Method 2” used data from the 8 wave probes deployed in the Octoprobe, and related the 1st harmonic phase difference between pairs of probes a known distance apart to the two unknowns, namely, local direction of wave propagation and absolute wave celerity. If the flow field was homogeneous, it would then be possible to produce 7 independent estimates of phase difference and, adopting an appropriate numerical technique, solve for celerity and direction. However, the present tests specifically included rapidly varying currents, and estimates predicted a possible 15% variation in celerity along the length of the Octoprobe. In order to improve the spatial

resolution, it was decided to analyse the data as two separate cross-shaped groups - with 5 probes in each group. Adopting a least-squares optimisation, it was then possible to deduce two independent sets of results for each deployment position. While this method was restricted to identifying the dominant propagation direction at a single incident wave frequency, it was found to be robust, made no assumptions concerning the local wave celerity, and produced reliable results for all test conditions investigated.

“Method 3” looked at the variation in phase of the 1st harmonic along a shore-normal transept as measured by the four co-linear wave probes on the spine of the Octoprobe as it was traversed across the jet current (**fig.2**). At any particular location the rate of change of phase is related directly to the component of celerity in the shore-normal direction. Then, making the (initially unsupported) assumption that the local absolute celerity can be calculated from the incident wave celerity and the local mean velocity (as measured by the ADV probes) using simple Doppler shift, it is possible to calculate the angle of incidence between the jet current and the local direction of wave propagation, in other words, the wave refraction. While this method worked well for the case with waves following the current, it failed to yield solutions for the opposing current case and has not been applied further.

The next (“Method 4”) used data from the Octoprobe to produce full directional spectra, providing both the directional distribution of wave energy at the incident wave frequency and the distribution between different frequencies at positions across the jet current. At each deployment location, analysis was performed on two groups of wave probes (with 4 probes in each) to improve spatial resolution. Spectra were calculated using the Iterative Maximum Likelihood Method (Pawka, 1983), adapted to include a dispersion relation varying with direction and local jet current strength. Implicit in this modification is the assumption, used in “Method 3”, that the local wave celerity can be calculated from a simple Doppler-shift of celerity from linear wave theory using the local mean current velocity.

Finally, from the data recorded, it would be possible to calculate the wave parameters using collocated measurements of water surface elevation and wave-induced velocities. However, time has so far precluded such analysis.

Results

Preliminary observations with no waves present confirmed that the jet current took on the desired Gaussian horizontal profile, but a gradual increase in width of the jet was noted as it propagated along the basin. However, this change was sufficiently weak to be deemed negligible along the 10m length of basin in which detailed velocity measurements were made. Some meandering of the jet was also detected in the time-series of u and v velocity components, and spectral analysis confirmed

oscillations between 40s and 100s, suggesting streamwise length scales of between 10m and 25m.

With the current turned off, waves of 0.8s period were then propagated shore-normal and at $\pm 30^\circ$ to shore-normal to assess both the quality of the waves and the reliability of the measuring and analysis techniques. Time-series of water surface elevations showed that the waves remained regular, with wave heights constant to 5% and no modulation. Spatial variation of wave height suggested reflection coefficients from the beach of less than 5%. The angle of wave propagation was found to be determined to within $\pm 1.5^\circ$ of the direction requested at the wave maker using data from the Octoprobe, with slightly poorer estimates from the velocity vector method.

When the waves were propagated across the jet current, the first notable observation was that the jet current had itself been moved laterally - inshore and away from the wave generators by 0.3m when the waves followed the current, offshore and towards the wave generators by 1.4m when the waves opposed the current. This change can be seen in the reference current plotted at the bottom of most subsequent diagrams, and changes in wave properties should be viewed relative to that location.

Fig.3 shows the local wave celerity and angle of propagation measured by the Octoprobe using "Method 2" when waves at 30° incidence propagate across the following jet current. The 20% increase in celerity observed at the centre of the jet agrees well with simple 1st order Doppler-shifted theory as used for gradually varied flows (Jonsson, 1990). However, the direction of propagation varies sinusoidally across the jet (oscillating about the predicted refraction pattern), with almost no refraction measurable at the centre and with 6° shifts in the two shear layers on either side. IMLM analysis ("Method 4") produced very similar results. In contrast, the propagation angle deduced from the velocity vectors (**fig.4**) shows rather higher angles of refraction - again varying sinusoidally across the jet but phased differently from the results reported above - with a 15° shift in the offshore shear layer. Despite the spatial variation in refractive angle, the scale is close to the 6° predicted from Snell's Law for slowly varying flow.

Figs.5 & 6 show the corresponding refraction patterns when the waves propagate into the opposing jet current. In this case the 20% decrease in celerity predicted by theory is again well matched to data from the Octoprobe, as is the refracted angle of propagation at the centre of the jet. However, inshore of the jet, the angle of refraction is erratic. The shift in propagation direction towards shore-normal calculated at the centre of the jet from the velocity vectors (particularly using ADV2) is significantly greater than found using the other methods or predicted by gradually-varied theory.

With the waves propagating orthogonally across the jet current, no significant

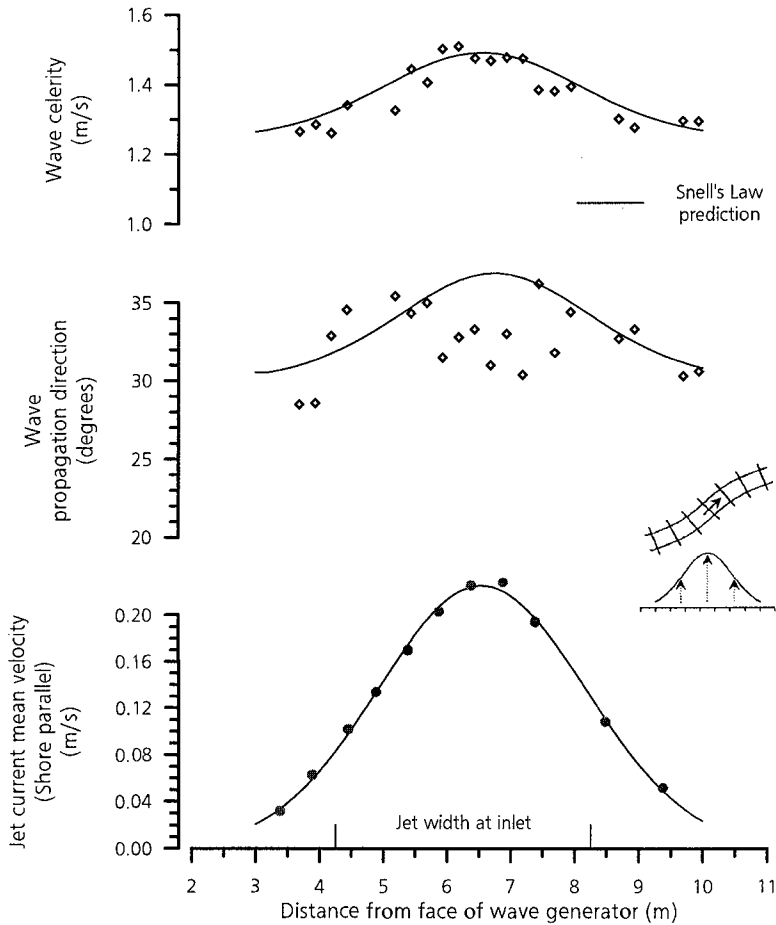


Figure 3: Variation of wave celerity and direction (from Octoprobe) across the jet current: following current case.

changes in celerity or angle of propagation were observed, although the scatter in all data sets was greater than observed in the absence of the current.

Figs.4 & 6 also show the variation in wave height measured across the jet current for the following and opposing current cases respectively. It can be seen that, within the bounds of experimental scatter, the significant wave height indicates a

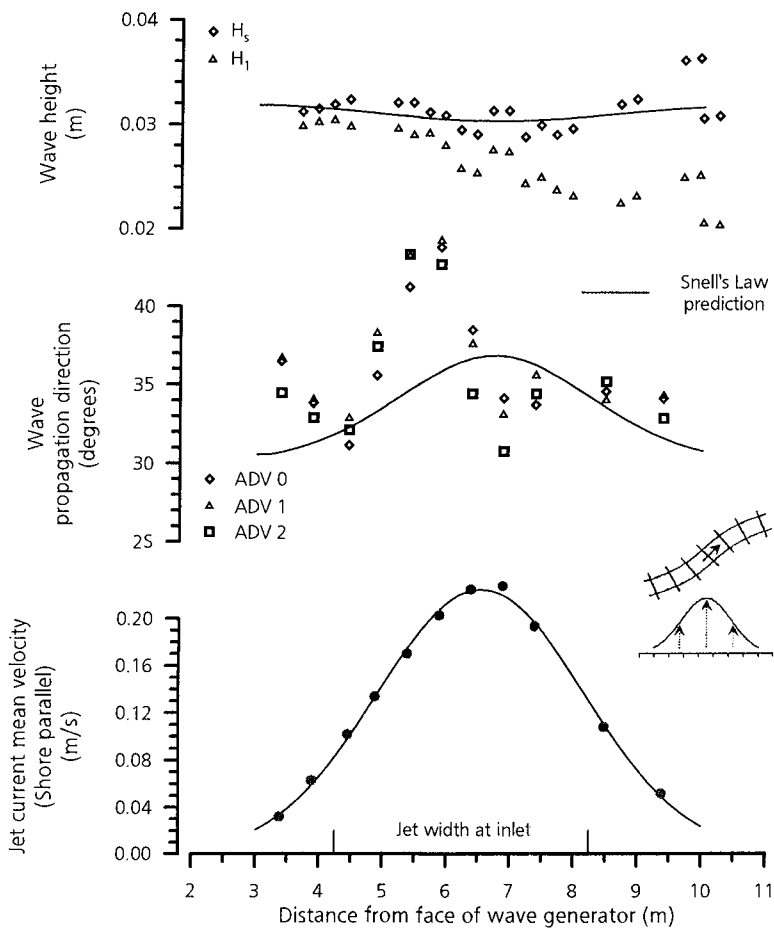


Figure 4: Variation of wave height and direction (from velocity vectors) across the jet current: following current case.

partial standing wave pattern (<10%) superimposed on the weak variation in H_s predicted by theory as the waves propagate across the jet. In contrast, the graph of 1st harmonic wave height H_1 , confirms the reflection pattern but suggests that H_1 decays rapidly across the jet. This apparent anomaly is caused by different methods

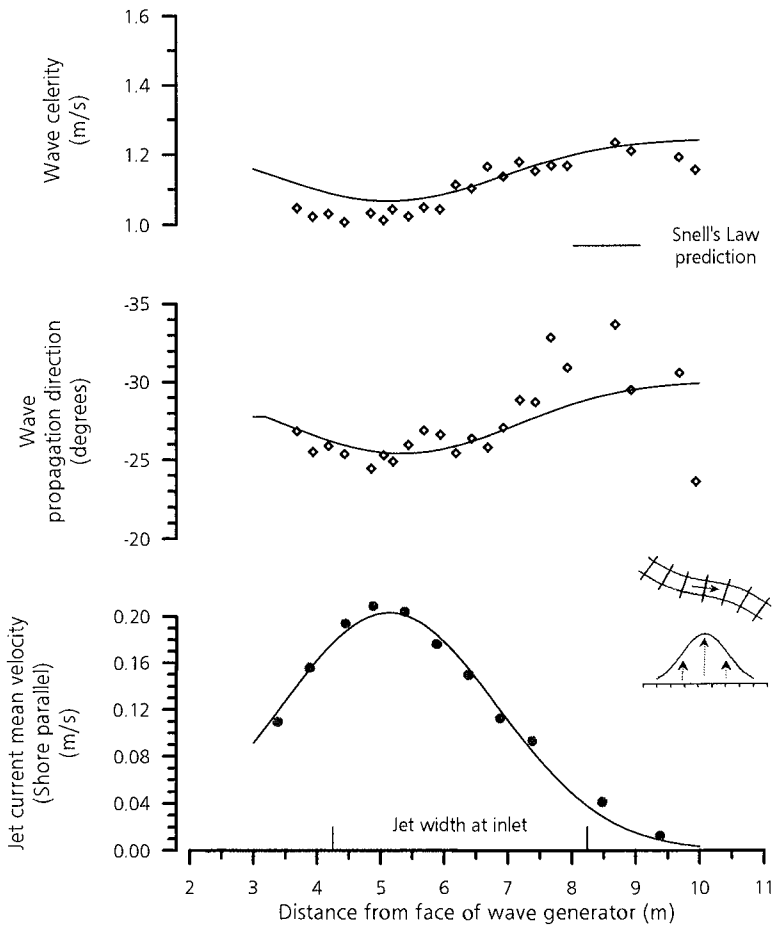


Figure 5: Variation of wave celerity and direction (from Octoprobe) across the jet current: opposing current case.

of analysis: H_s has been calculated from the standard deviation of the wave surface time-series, whereas H_l comes from Fourier analysis of the ensemble-averaged surface profile (phase-locked to one of the offshore fixed wave probes). The problem arises because wave time-series records on the inshore side of the jet current exhibit

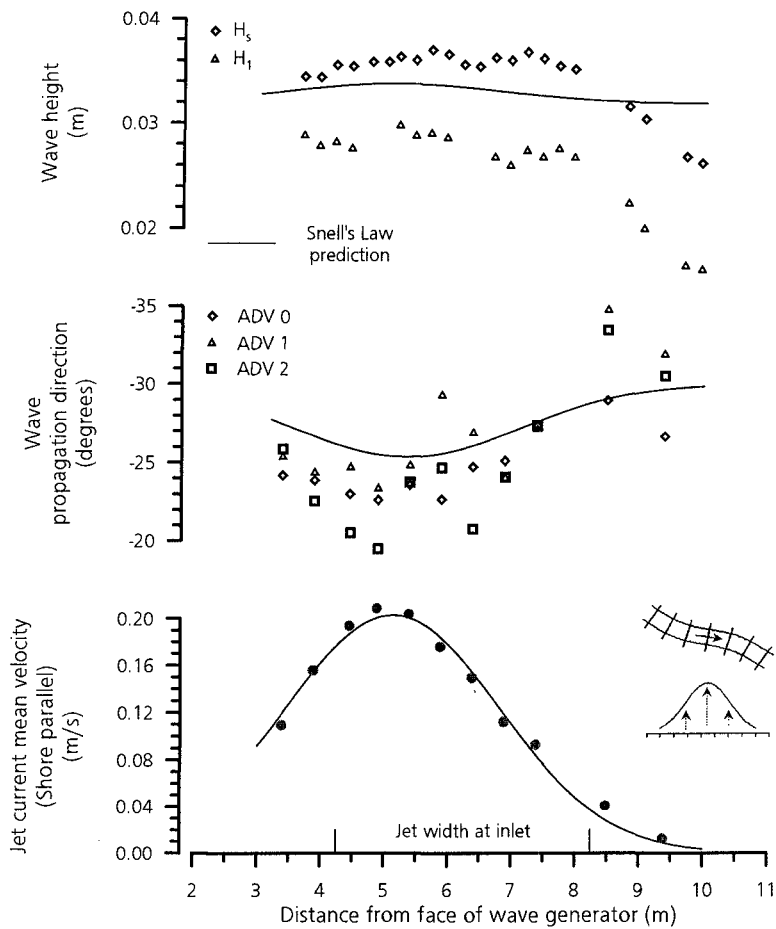


Figure 6: Variation of wave height and direction (from velocity vectors) across the jet current: opposing current case.

modulation patterns; these cause the ensemble-averaging process to move in and out of phase, thus reducing the amplitude of the resulting ensemble average and of the wave harmonics calculated from it. Clearly, in this situation, the significant wave height H_s is giving the true picture as far as wave height change across the jet is

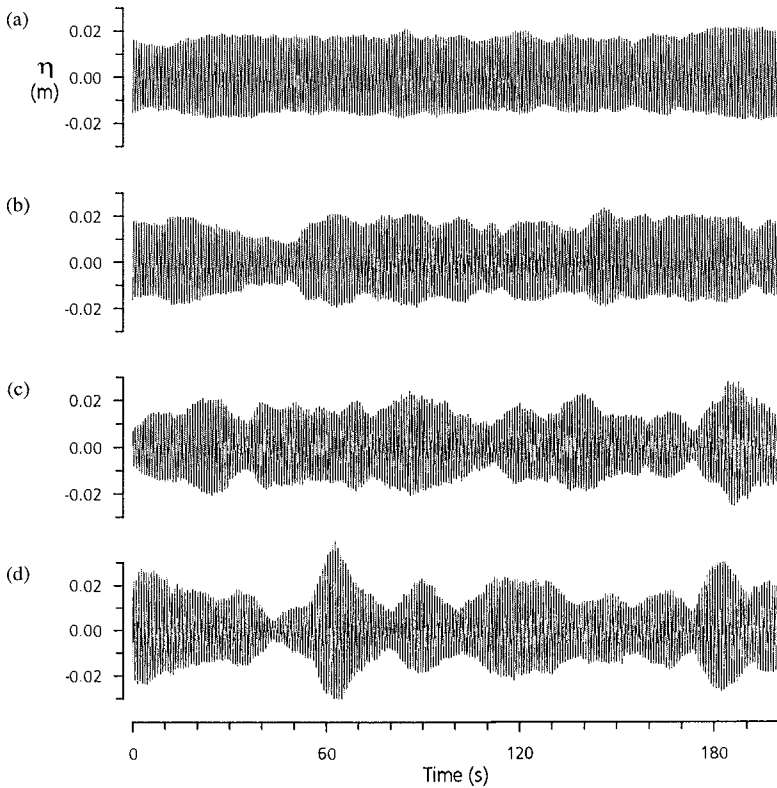


Figure 7: Surface elevation time series at locations across the jet current for the wave following current case. (a) 3.4m, (b) 5.4m, (c) 7.4m, (d) 9.4m.

concerned, and the harmonics should be ignored.

The modulation in wave amplitude referred to above is an interesting phenomenon which needs to be considered when interpreting the results from this study. Time-series of wave surface elevation recorded at various locations across the jet current (**fig.7**) show that while there is some modulation at the offshore edge, the effect is worse inshore, where the waves have already propagated across the jet for some distance. These features generally exhibit time-scales of approximately 30s, which is rather lower than those noted above in relation to the meandering of the jet current.

While it has not yet been possible to model the observations in detail, it is

clear from the descriptions above that the angles of propagation (**figs.3-6**) observed on the jet can be explained as the effect of reflections (<10%) from the jet-current shear layers superimposed on the refraction angle predicted by gradually-varied flow theory. The phase of these reflected waves will vary depending on the exact location of the jet current at the point of reflection; similarly, the phase of the incident waves will depend on the location of the jet at the point where they first enter the shear layer. Hence the wave modulation will be related to the meandering of the jet but will not necessarily correlate with the time variation at any one point along the basin. The resultant wave pattern on the jet will thus be a function of incident wave characteristics, reflection coefficient from the shoreward shear layer, possible secondary internal reflection from the offshore shear layer, the mean horizontal jet current profile, and the temporal and spatial scales of the current meandering.

Conclusions

A significant data set has been acquired, providing information on the transformation of regular waves propagating across a narrow jet-like current with strong horizontal shear. Four methods have been used to measure the refraction of waves on the spatially varying current. One of these methods has been able to provide an independent measure of the local wave celerity on the jet current, and this confirms the predictions from a simple Doppler shift of linear wave theory.

The apparent angle of propagation of the incident waves does not follow a simple variation across the jet, but has been shown to be sensitive to the effect of small waves reflected from the shear layers.

In the absence of the jet current, wave amplitude is constant with time. When the current is added, the wave amplitude becomes modulated by the interaction of the incident waves with reflections from the gradually-meandering shear layer.

When waves are propagated across the jet-like current in a large wave basin such as the UKCRF, the jet is moved laterally. For following waves it translates towards the shoreline; for opposing waves it translates away from the shore.

Acknowledgements

This work was funded by the Commission of the European Communities Directorate General for Science, Research and Development under Contract MAS3-CT95-0011 as part of the KADWCI project: *Kinematics and Dynamics of Wave-Current Interactions*. The authors also wish to express their gratitude to HR Wallingford for allowing use of the UK Coastal Research Facility, and in particular to Adrian Channell for his support in ensuring that the tests were completed on time.

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