Experimental Study of Wave Breaking and Blocking on Opposing Currents

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Abstract

A series of experiments on monochromatic waves being blocked by opposing currents are presented in this paper. A heirarchial pattern in the wave field characteristics was observed with increasing wave amplitude. A shoaling model using a bore dissipation formulation to simulate current limited breaking is compared with the data. It is observed that amplitude dispersion plays an important role in wave blocking and cannot be ignored. For the large amplitude waves side band instabilities radically affect the wave field and prevent wave blocking.

Introduction

Wave blocking occurs in regions (e.g. mouths of river inlets) where the currents are opposing the waves. The opposing current slows down the waves leading to an increase in the wave steepness, which sometimes leads to wave breaking. The waves get blocked when the current is strong enough to prevent the wave energy from traveling upstream. That is when the group velocity C_g goes to zero. The steep waves in front of the blocking point cause considerable navigational hazards and can also effect sediment transport. Since the wave field is dramatically altered through the blocking region, it becomes important for the coastal engineer to be able to quantify under which conditions wave blocking will occur. There is also a limited understanding of energy dissipation due to current limited wave breaking. Thus, from an engineering view point a study of wave blocking and the evolution of the wave field through these blocking points is very important.

Before progressing into the dynamics of wave blocking, it is important to know the conditions under which wave blocking occurs. Consider a two dimensional wave moving on a depth-uniform current U. Then in a frame of reference moving with the current, the equations and solutions for water wave motion are identical to the case of no current. If ω is the wave frequency in a stationary reference frame, and σ is the wave frequency in a reference frame moving with the current, then the phase speed of the wave in the moving reference frame can be related to the phase speed in a

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stationary reference frame by

$$\frac{\omega}{k} = \frac{\sigma}{k} + U \tag{1}$$

or

$$\omega - kU = \sigma \tag{2}$$

where k is the wave number and remains unchanged in the two reference frames. The expression for σ depends upon the theory used to obtain the dispersion relation from the equations. Differentiating (2), and setting $C_g = \frac{\partial \omega}{\partial k}$ to zero we then get the condition for wave blocking as

$$\frac{\partial \sigma}{\partial k} = -U \tag{3}$$

From (3) it is obvious that wave blocking can only occur if the wave and current are moving in opposite directions.

A linear uniform asymptotic solution for the waves close to the blocking point was first obtained by Smith (1975), who showed that, for small wave amplitudes, the waves are reflected from the blocking point. These reflected waves get shorter and shorter as they move away from the blocking point. Stiassnie and Dagan (1979) showed that when the required blocking current is close to the maximum current then partial wave reflection occurs. Their solution provided the transition region from no reflection to total reflection of wave energy. Some nonlinear aspects of the waves at the blocking points were also studied by Peregrine and Smith (1979). Both Shyu and Phillips (1990) and Trulsen and Mei (1993) further enhanced the theory of wave blocking by including the effects of surface tension. They showed the existence of a second blocking point which occurs when the reflected waves get so small that surface tension effects become predominant in the dispersion relation.

All the references cited above have concentrated on the study of wave reflection from the blocking point. But wave reflection occurs only if the initial wave amplitude is fairly small. For most practical problems the waves become steep enough to break as they approach the blocking point with very little to no wave reflection. The process is complex and requires an extensive experimental study. Unfortunately there are very few references in the literature to experimental studies of wave blocking. Lai *et al.* (1989) have conducted laboratory studies of the kinematics of the wave-current interaction under blocking conditions but do not discuss any of the dynamics. Sakai and Saeki (1984) have also conducted experiments of waves on opposing currents, but their experimental setup also includes a sloping sea bed. The focus of their study was the combined effect of opposing currents and sloping sea bed on wave transformation and breaking. In their experiments wave breaking occurs in shallow water, and it thus becomes difficult to isolate current limited wave breaking from depth limited wave breaking. There are still a number of unanswered questions, includingl, what are the factors affecting wave blocking, how do waves break and dissipate energy under strong opposing current, and how do the wave field characteristics change with increasing nonlinearity. Therefore, we need further experimental studies to get a better understanding of the evolution of the wave field through the blocking region and

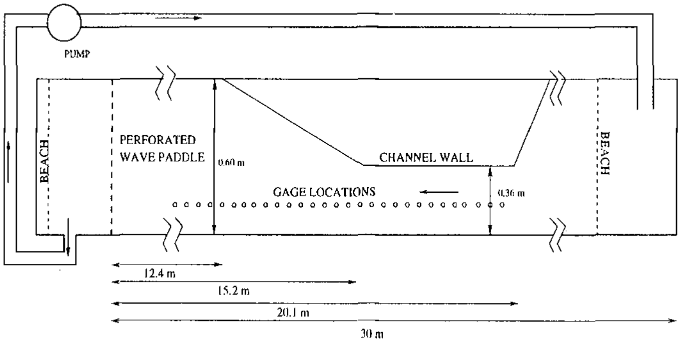

Figure 1: Schematic Plan view of the experimental setup

the dynamics of the wave-current interaction. Keeping this aim in mind the present study was carried out. A series of experiments have been conducted with monochromatic waves to provide some useful insights into the characteristics of the wave field in blocking currents, and the conditions under which the waves get blocked. The experiments have been conducted in relatively deep water so as to minimize the interaction between the bottom bathymetry and the waves. A simple shoaling model for waves on currents combined with a bore dissipation model to simulate wave breaking has also been compared with the data. This has been done with an attempt to quantify current limited wave breaking.

Experimental Setup

The experiments were conducted in the recirculating flume at the Center for Applied Coastal Research. A schematic plan view of the setup is shown in Figure 1. The flume is 30 m long and 0.60 m wide. All the tests were conducted in a water depth of $h = 0.50$ m. Currents are generated with the help of a pump which removes the water from behind the wavemaker and puts it back at the far end. A narrow channel has been created with the help of a false wall so as to be able to generate faster currents in the middle of the flume. A perforated wave paddle is used to generate waves so that the current can be allowed to go through the paddle. Porous beaches have been placed at the two ends of the flume to absorb the waves. All measurements in the $x-$ direction are made from the still position of the wave paddle.

Data was recorded in the form of a time series of the water surface at 29 different locations extending through the channel. Capacitance-type wave gages were used for this purpose. The gage locations are shown by open circles in Figure 1. At each location data was collected for 600 wave periods and wave height estimates obtained using a zero-up crossing method. The current field was measured using an acoustic doppler velocimeter. Figure 2 shows the vertical profile of the mean current (averaged over 330 seconds) at 5 different locations in the flume. Due to the development of

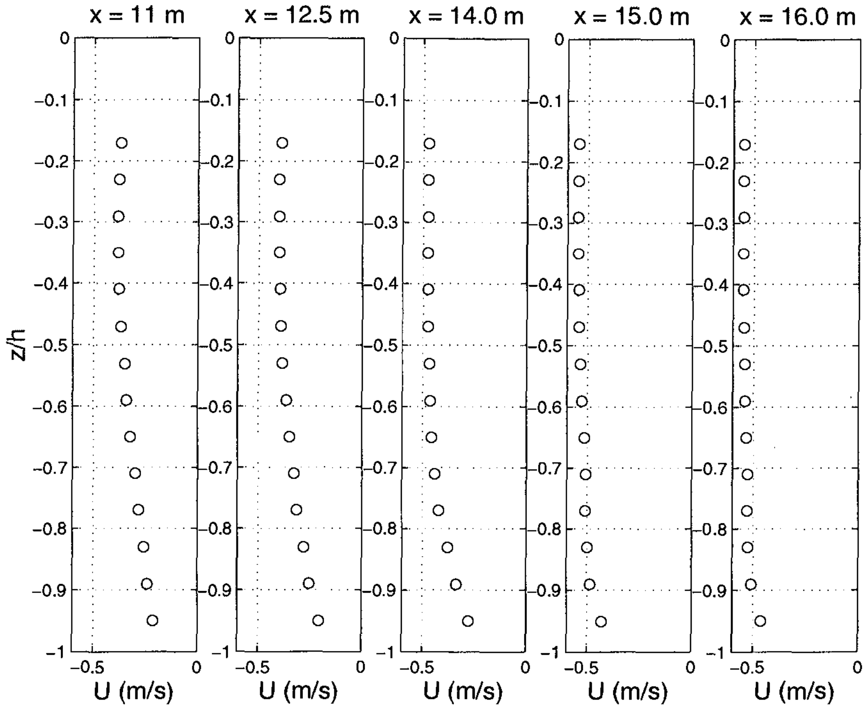

Figure 2: Vertical distribution of the current away from the channel and inside the narrow channel

a bottom boundary layer there is a slight shear in the current profile, specially in the region closer to the wavemaker. But inside the channel the current is more or less depth uniform. Thus, for the remainder of the paper we shall consider a depth averaged value of the current only. As a result of the constriction caused by the narrowing of the channel, the depth averaged current varies from 0.32 m/s at $x = 12.4$ m to its maximum value of 0.53 m/s at $x = 15.2$ m.

Various tests were conducted of which 6 cases will be shown here. The particulars about the tests are given in Table 1. In these cases the blocking point (according to linear dispersion) for the given wave period occurs close to the narrow part of the channel ($x = 14.6$ m). The incident wave amplitude was increased from test 1 to 6 and the characteristics of the wave field observed.

Theoretical Model

Table 1: Test Particulars

Test	T (sec)	H_o (cms)
1	1.2	1.223
2	1.2	1.8
3	1.2	3.344
4	1.2	6.607
5	1.2	9.512
6	1.2	12.61

Bretherton and Garrett (1969) showed that in the presence of currents it is the wave action that is conserved and not the wave energy. Their conservation principle is given by

$$\frac{\partial}{\partial t}\left(\frac{E}{\sigma}\right) + \nabla . \left(\frac{E}{\sigma} C_g \frac{\vec{k}}{k}\right) = 0 \tag{4}$$

where $\frac{E}{\sigma}$ is the wave action and E is defined by linear theory as

$$E = \frac{1}{8}\rho g H^2$$

H being the wave height.

Since we are considering only steady wave conditions, we can neglect the first term in (4). Allowing for the variation of the channel width, integrating across the channel and providing a dissipation function for wave breaking, we get

$$\frac{1}{b}\frac{\partial}{\partial x}\left(\frac{bEC_g}{\sigma}\right) = \frac{D'}{\sigma} \tag{5}$$

where b is the channel width. D' is defined according to the bore dissipation model of Battjes and Janssen (1978), and is given by

$$D' = \frac{-\beta}{\pi}\left(\sqrt{\frac{8}{\rho h}}\right) k E^{3/2} \tag{6}$$

where β is a parameter. To match this dissipation rate with our data we take $\beta = 0.20$. It should be kept in mind that this is ten times lower in magnitude than the value used for modeling waves breaking on a sloping beach (Battjes and Janssen, 1978). This is due to the fact that current limited wave breaking is very different from depth limited wave breaking even though we are using the same formulation to model the two processes. Figure 3 shows a picture of waves breaking on the current. From the picture we can see that the breaking is weak and unsaturated as opposed to the saturated breakers observed in depth limited breaking.

The onset of wave breaking is predicted by Miche's criterion given by

$$H_m = \frac{\alpha}{k}\tanh kh \tag{7}$$

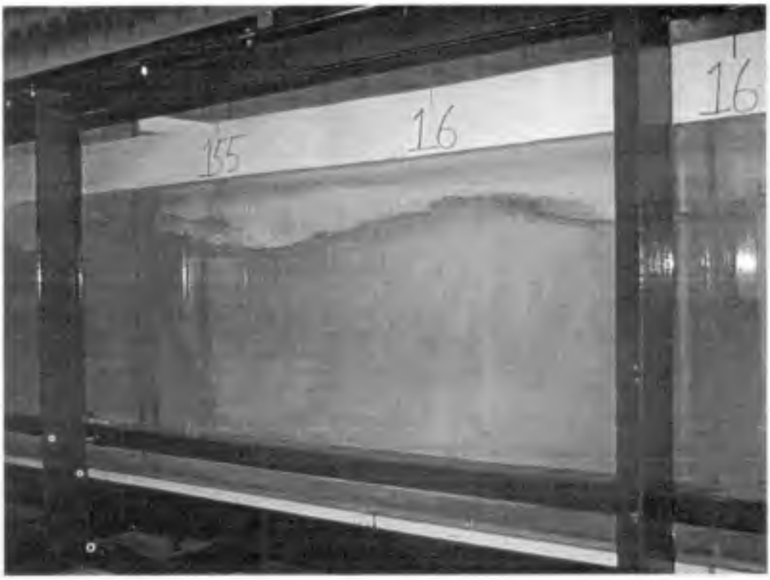

Figure 3: Waves breaking on the current (Test 6)

where α is a parameter. By taking the Stokes wave solution and assuming that at the onset of breaking the fluid velocity at the crest of the wave is equal to the phase velocity, Miche got $\alpha = 0.88$. This value of α has been used in the literature to predict the onset of depth limited breaking (Battjes and Janssen, 1978). However, based on our observations α has a lower value ($\alpha = 0.60$) for current limited wave breaking. Sakai and Saeki (1984) have shown through their experimental results that $\alpha = 0.88$ works reasonably well when there are no opposing currents, but it decreases with increase in the opposing current. The smallest value of α that they got was approximately 0.7. It should be kept in mind that their experiments were carried out in shallow water ($kh \approx 0.63$) where both depth and current play a part in wave breaking, while our experiments were carried out in much deeper water ($kh \geq 2.4$) where the wave breaking is caused only by the opposing current.

Eq. (5) together with (6) gives a model for a wave shoaling and breaking on a current . The model does not account for wave reflection, and thus cannot be used in cases where the waves are reflected from the blocking point. Also, it can be seen from (4) that at the blocking point ($C_g = 0$) the model becomes singular. Thus the model can only be used until the blocking point.

The dispersion relation is given by (2) together with an expression for σ which depends upon the wave theory used. According to linear wave theory we have

$$\sigma = \pm\sqrt{gk \tanh kh} \tag{8}$$

and according to a third order Stokes wave theory (Bowden, 1948) we have

$$\sigma = \pm\sqrt{gk \tanh kh \left[1 + (ka)^2 \left(\frac{8 + \cosh 4kh - 2\tanh^2 kh}{8\sinh^4 kh}\right)\right]} \tag{9}$$

To determine the importance of amplitude dispersion in wave blocking, the model shall be compared with data using both the dispersion relations. From (3) and (9) we can already see that amplitude dispersion tends to increase the required blocking current, by increasing the relative wave frequency σ at a fixed k.

Observations

Out of the 6 test cases, only in test 1 was the incident wave height small enough for the waves to be reflected from the blocking point. Since the gages were too far apart to obtain the envelope of the partial standing waves, the experiments were repeated for this case with the gages densely spaced between $x = 13.7$ m and $x = 16$ m.

Smith (1975) and all subsequent references in the literature on wave reflection from a blocking point have shown that the envelope of the waves through the blocking point is represented by an Airy function, within the context of the linearized theory. From Trulsen and Mei (1993), the wave amplitude near the reflection point is given by

$$a = b_0 Ai(-\alpha^{2/3}(x - x_{st})) \tag{10}$$

where, x_{st} is the location of the blocking point (for our test case $x_{st} = 14.6$ m), b_0 is a constant related to the incident wave amplitude, and α is given by

$$\alpha = \sqrt{\left(\frac{-2\frac{\partial U}{\partial x} k\sigma}{3U^2}\right)\Bigg|_{x=x_{st}}} \tag{11}$$

Matching the data to (10) at $x = 13.7m$ and then comparing the amplitude envelope to the Airy function (Figure 4) we find that (10), which is based on linear theory, gives a good estimate of the actual shape of the amplitude envelope but underestimates the first peak. The accentuation in the wave envelope occurs because the larger wave amplitude close to the blocking point makes nonlinear effects important. The nonlinear term in a cubic Schrodinger equation for a modulating wave-train changes sign at $kh = 1.36$, leading to a self-focusing modulation for $kh > 1.36$, and a defocusing modulation for $kh < 1.36$. The result of the self-focusing mechanism, expected to be dominant here, is to accentuate the height of envelope modulations. A similar effect occurs in the case of an evolving sinusoidal wave front, where in deep water, linear theory again underestimates the first peak of the amplitude envelope in the front (Mei, 1992, page 57, Figure 4.2). It should be noted that these are just

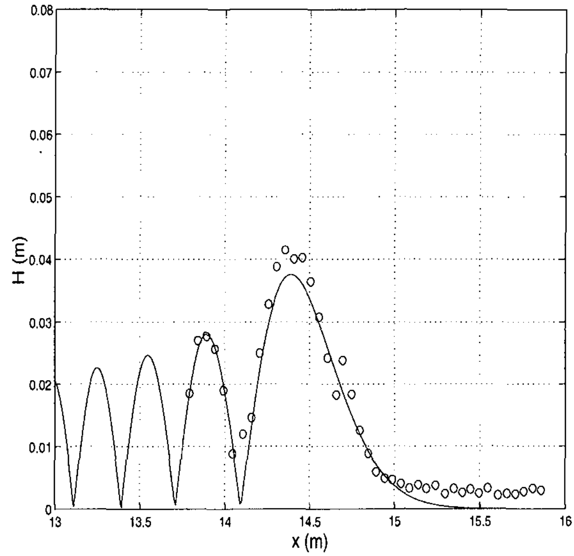

Figure 4: Amplitude envelope of partial standing waves due to reflection from the blocking point. 'Solid line' Airy function. 'Circles' Data

the preliminary findings and a thorough analysis of wave reflection from the blocking point is currently being carried out.

Tests 2 to 6 all have wave breaking with no apparent wave reflection. We can thus compare the data with the numerical model. The comparison is shown in Figure 5. In each of the test cases the wave heights have been normalized by their corresponding incident wave height. The depth averaged velocity distribution is also shown as a function of x. In the case of test 2 the wave shoals as the opposing current is increased and gets blocked before it can reach the maximum limit of the current. Test 3 shows a similar trend but now the blocking point is shifted back further from the wave paddle due to amplitude dispersion caused by the larger wave height. Wave blocking now occurs at the maximum current. In test 4, due to an even larger wave height, the wave reaches the maximum current without getting blocked. At this point, if the wave height does not reduce, the wave will travel right through the channel. But the wave is breaking and thus continues losing energy until the magnitude of the current in the narrow channel is enough to block it. In these three cases we find that the model works very well in predicting the blocking point when using Stokes third order dispersion as opposed to linear dispersion. The model also performs better in estimating the shoaling of the waves before blocking when taking amplitude dispersion into account.

Tests 2 to 4 are similar in the sense that the waves break and get blocked. But

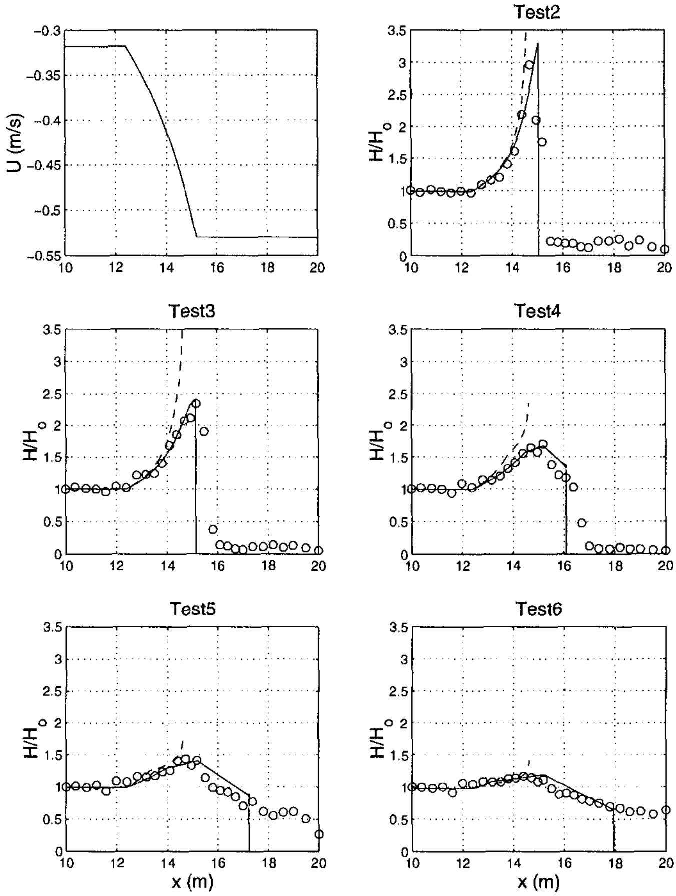

Figure 5: Model to data comparisons of the wave height distribution. 'Solid line' Bore model with Stokes third order dispersion, 'dashed line' Bore model with linear dispersion, 'circles' Data

tests 5 and 6 show a different pattern. From Figure 5 it can be seen that in both cases the waves reach the maximum current, and just as in test 4, propagate into the channel and continue dissipating energy due to wave breaking. However, the waves do not get blocked even when the blocking condition is satisfied for the model. Why? To get a better idea let us consider the wave period distribution as a function of x for the different test cases. From Figure 6 it can be seen that for tests 5 and 6 the wave energy shifts to a lower frequency as it propagates through the channel. For these

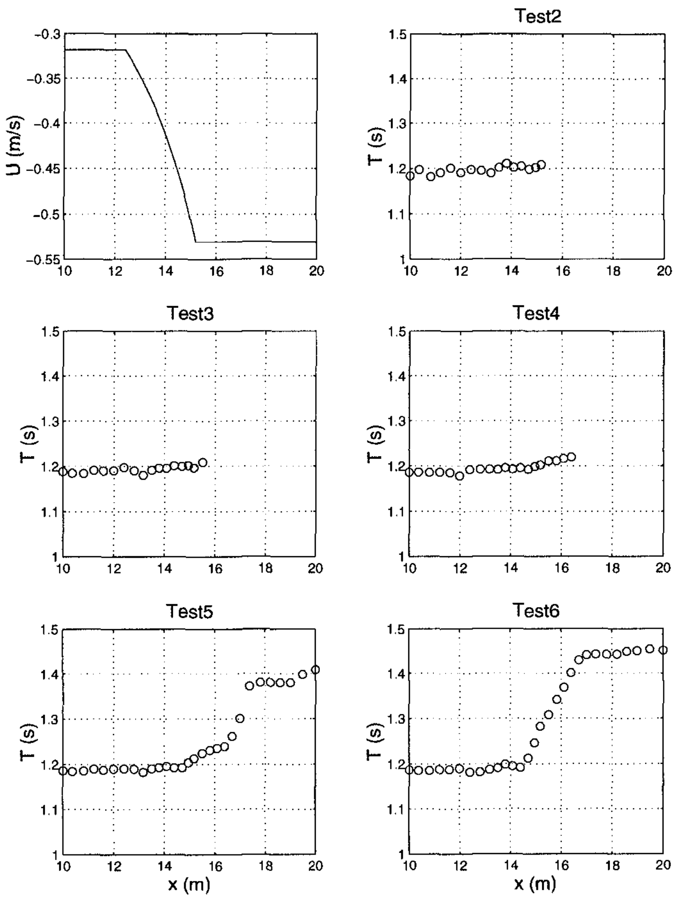

Figure 6: Significant wave period at the different data points obtained by the Zero-upcrossing method. Clear shift in the energy to a lower frequency for Tests 5 and 6

lower frequencies the required blocking current (even in the linear limit) is greater than the maximum current and the waves do not get blocked. The model, on the other hand, is unable to reproduce the shift to lower frequencies and thus predicts that waves are blocked.

The shift in the wave energy to a different frequency is explained by the growth of lower side bands. Benjamin and Feir (1967) showed that in deep water, steep waves

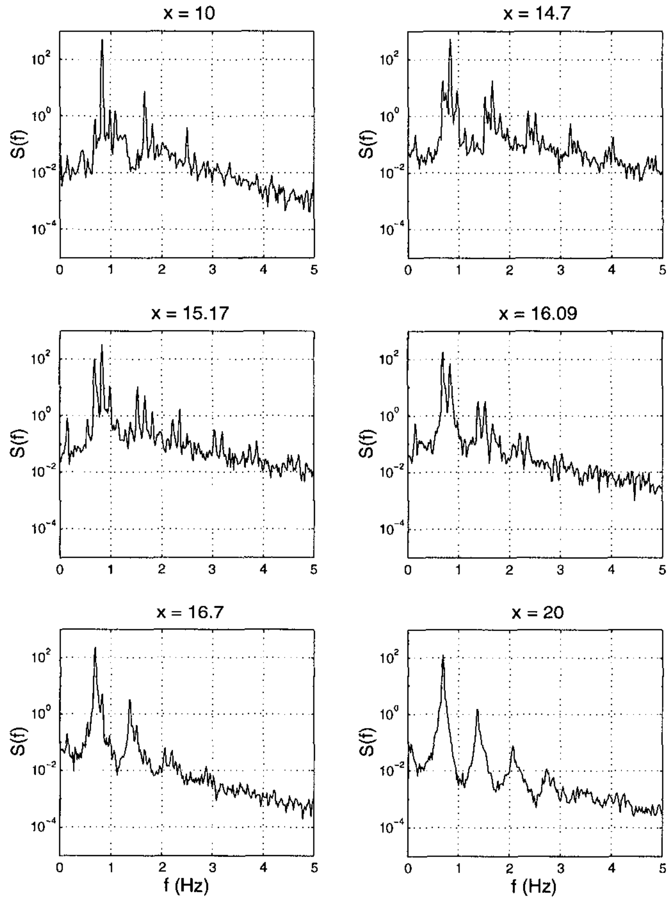

Figure 7: Frequency Spectra for Test 6 at various locations in the channel. Energy is transferred to the lower side band while the primary wave is blocked.

are unstable to disturbances at some frequencies δf away from their fundamental frequency (where δf is a small number). This leads to the growth of bands of energy on either side of the fundamental frequency in the frequency spectra, which are referred to as side bands. The growth of these side band instabilities depends upon the wave steepness and water depth. They become highly pronounced when the waves are riding on opposing currents and have been observed in the laboratory (Lai *et al.*, 1989). As the waves approach the blocking point the group velocity C_g tends to zero and the wave energy travels very slowly. Thus, a significant amount of energy could be transferred from the fundamental frequency to the side bands even through small spatial distances, as the time of passage of a given wave energy packet through the transition region is relatively long. Since the lower side band requires a stronger blocking current than the primary wave or the upper side band, it could then continue to propagate forward while the other two are blocked.

This is clearly evident in test 6. The frequency spectra at six different gage locations are shown in Figure 7. At $x = 10$ m the wave is away from the channel but the side bands are clearly visible. The fundamental frequency is $f = 0.833Hz$ while the most prominent lower and upper side bands occur at $f = 0.688Hz$ and $f = 0.978Hz$ respectively. As the waves propagate into the channel the growth of these side bands is clearly evident. Going from $x = 14.7$ m to $x = 15.17$ m the energy in the lower side band increases by almost 10 times in less than 0.5 m. In the narrow part of the channel, first the upper side band and a little later the primary wave get blocked. But the wave energy continues to propagate through via the lower side band. A plot of the corresponding time series (Figure 8) shows the shift to a longer wave period. During the transfer of energy to the longer wave period the waves become very groupy. This increases the complexity of the breaking process as the waves tend to break at the crests of the wave groups. A similar pattern was also seen in test 5. Tests 2 to 4 showed no side band instabilities due to the smaller wave steepness.

Conclusions

In this work we have studied a series of monochromatic waves on an opposing blocking current. The wave amplitudes considered varied from small to large. For the smallest wave amplitudes, wave reflection from the blocking point was observed. Preliminary results show that though the amplitude envelope confirms well with the theoretical predictions, discrepancies between the data and theory show up close to the blocking point due to nonlinear effects.

A simple wave action conservation model together with a bore dissipation breaking model is used to simulate the data. Comparisons show that amplitude dispersion plays an important role in wave blocking. A third order Stokes dispersion relation works much better than a linear dispersion relation in predicting the blocking point. Also the predictions of energy decay from the bore model are reasonably accurate upto the blocking point.

For the largest wave amplitudes the wave energy shifts to a lower frequency due to side band instabilities and the waves do not get blocked, while the model predicts wave blocking. The growth of side bands also makes the waves very groupy, and thus in turn increases the complexity of wave breaking. It is clear that side bands in a carrier wave play an important role in determining whether waves are blocked or not, and ignoring them could lead to significant errors in wave modeling. This phenomenon requires further study and efforts are underway to try and predict the growth of these instabilities using a Schrodinger's equation.

A hierarchy in the wave field characteristics has been observed as the wave amplitude is increased. Depending upon the initial wave amplitude and frequency a monochromatic wave on an opposing current could

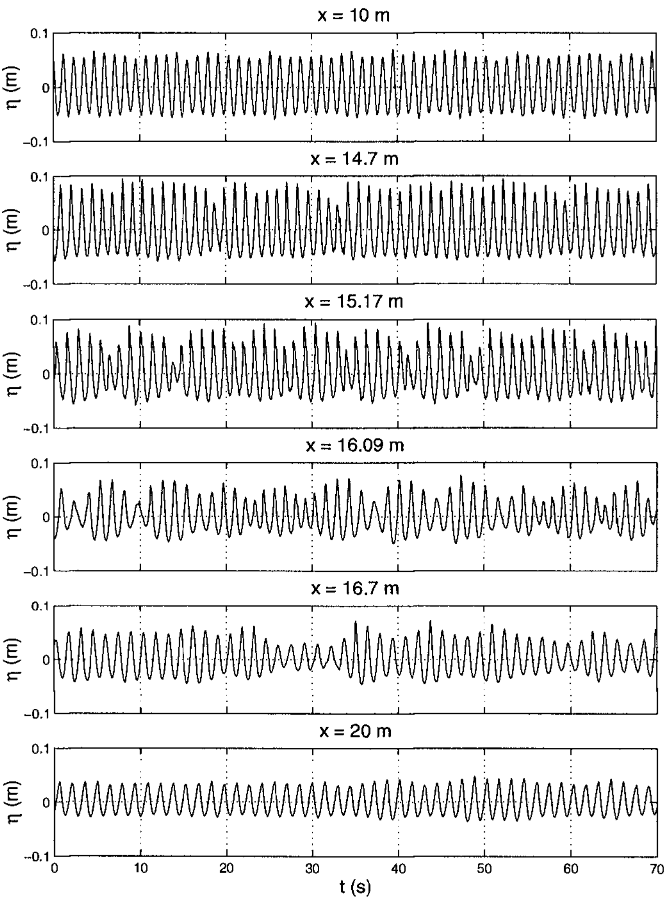

Figure 8: Time series for Test 6 at various locations in the channel.

1. be blocked and reflected,
2. be blocked and break at the blocking point,
3. pass through the maximum current due to amplitude dispersion but break and still get blocked, or
4. transfer significant energy to a lower side band which does not get blocked while the primary wave and the upper side band do.

In a random sea all these different phenomena could be occurring at the same time making dissipation modeling all the more difficult. Thus the next step is to study wave breaking in groupy and random waves.

Acknowledgments

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